correction TD T. Laplace

Exercice 1

$$F_1(p) = \frac{3p-137}{p^2+2p+401}$$

on cherche les racines du dénominateur

$\Delta = 2^2 - 4\times 401 = -800 < 0 \Rightarrow$ pas de racines réelles.

on remarque : $p^2+2p+401 = \underbrace{p^2+2p+1}_{(p+1)^2}+400$

donc $F_1(p) = \dfrac{3p-137}{(p+1)^2+400} = \dfrac{3(p+1)-140}{(p+1)^2+400}$

$$F_1(p) = 3\times\frac{p+1}{(p+1)^2+20^2} - 7\times\frac{20}{(p+1)^2+20^2}$$

donc $f_1(t) = TL^{-1}\left(3\dfrac{p+1}{(p+1)^2+20^2} - 7\dfrac{20}{(p+1)^2+20^2}\right)$

$$f_1(t) = 3\,TL^{-1}\left(\frac{p+1}{(p+1)^2+20^2}\right) - 7\,TL^{-1}\left(\frac{20}{(p+1)^2+20^2}\right)$$

$$f_1(t) = 3\,e^{-t}\cdot\cos 20t - 7e^{-t}\cdot\sin 20t$$

$$\boxed{f_1(t) = e^{-t}\left(3\cos 20t - 7\sin 20t\right)}$$

Exercice 2

$y'' + y' + 9y = 0 \quad (1)$

$y(0) = 0{,}2$ et $y'(0) = 0$

$$TL(y') = p\,TL(y(t)) - y(0)$$

$$TL(y'') = p^2\,TL(y(t)) - p\,y(0) - y'(0)$$

$TL(1) \longrightarrow p^2Y(p) - p\,y(0) - \underbrace{y'(0)}_{0} + pY(p) - \underbrace{y(0)}_{0,2}$
$+ 9Y(p) = 0$

$$p^2 Y(p) - 0{,}2\,p + p\,Y(p) - 0{,}2 + 9\,Y(p) = 0$$

$$\Rightarrow (p^2 + p + 9)\,Y(p) = 0{,}2(p+1)$$

$$\Rightarrow Y(p) = \frac{0{,}2.(p+1)}{p^2+p+9}$$

$\Delta < 0 \Rightarrow$ pas de racines réelles.

$$\Rightarrow Y(p) = \frac{0{,}2.(p+1/2) + 0{,}1}{(p+\frac{1}{2})^2 + \frac{35}{4}}$$

$$Y(p) = 0{,}2.\frac{p+1/2}{(p+\frac{1}{2})^2+\frac{35}{4}} + \frac{0{,}1}{(p+\frac{1}{2})^2+\frac{35}{4}}$$

$$Y(p) = 0{,}2.\frac{p+1/2}{(p+\frac{1}{2})^2+\frac{35}{4}} + \frac{0{,}1}{\sqrt{\frac{35}{4}}} \times \frac{\sqrt{35/4}}{(p+1)^2+\frac{35}{4}}$$

$$Y(p) = 0{,}2\,\frac{p+1/2}{(p+\frac{1}{2})^2+\left(\sqrt{\frac{35}{4}}\right)^2} + \frac{0{,}1}{\frac{1}{2}\times\sqrt{35}} \times \frac{\sqrt{35/4}}{(p+1)^2+\left(\sqrt{\frac{35}{4}}\right)^2}$$

donc $y(t) = 0{,}2\,e^{-\frac{1}{2}t}\cos\sqrt{\frac{35}{4}}\,t + \frac{0{,}2}{\sqrt{35}}\,e^{-\frac{t}{2}}\sin\sqrt{\frac{35}{4}}\,t$

$$\boxed{y(t) = e^{-\frac{1}{2}t}\left(0{,}2\cos\sqrt{\frac{35}{4}}\,t + \frac{0{,}2}{\sqrt{35}}\sin\sqrt{\frac{35}{4}}\,t\right)}$$