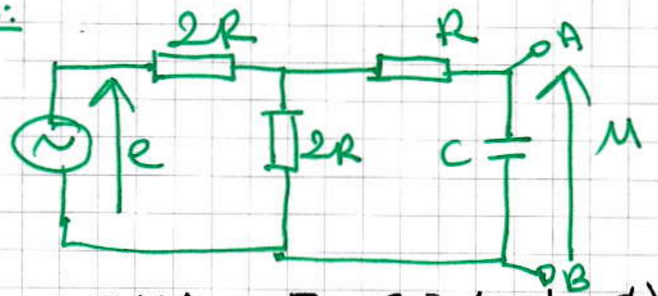
Correction TD filtrage

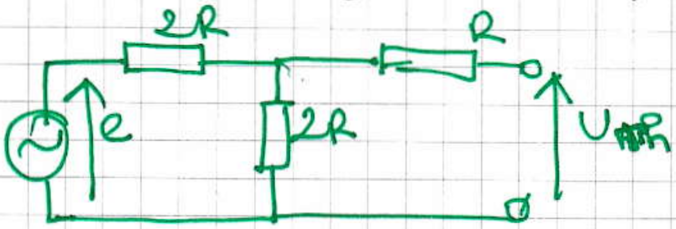
①

Exercice 1:



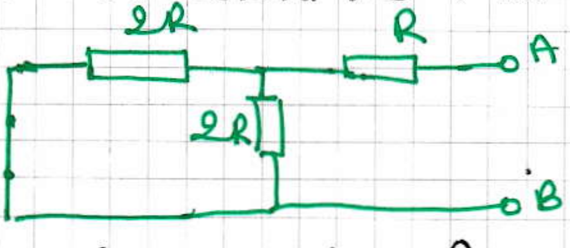
$$e(t) = E_0 \sin(\omega t + \phi)$$

1°/ on cherche le Modèle équivalent de Thévenin du circuit sans la capacité C.

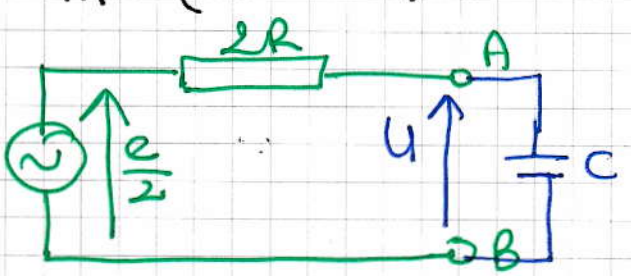


$$U_{Th} = \frac{e \times 2R}{2R+2R} = \frac{e \cdot 2R}{4R} = \frac{e}{2}$$

- calcul de R_{Th} vue des points A et B, en court-circuitant la tension $e(t)$.



$$R_{Th} = (2R \,||\, 2R) + R = R + R = 2R$$



on a notre M.E.T "en vert".

$$\underline{u} = \frac{e}{2} \times \frac{Z_c}{2R + Z_c} = \frac{e}{2} \times \frac{1/jC\omega}{2R + \frac{1}{jC\omega}}$$

$$\boxed{\underline{u} = \frac{e}{2} \cdot \frac{1}{1 + 2jRC\omega}}$$

loi des mailles :

$$\frac{e}{2} - 2R\,i - u = 0 \quad \text{avec } i = C\frac{du}{dt}$$

$$\frac{e}{2} - 2RC\frac{du}{dt} - u = 0$$

$$u + 2RC\frac{du}{dt} = \frac{e}{2} \quad \text{on pose } \boxed{\tau = 2RC}$$

$$\Rightarrow \boxed{u + \tau\frac{du}{dt} = \frac{e}{2}}$$

2°/ $u(t) = U_0 \sin(\omega t + \psi)$

on a $\underline{u} = \frac{\underline{e}}{2} \cdot \frac{1}{1 + j2RC\omega} = \frac{\underline{e}}{2} \cdot \frac{1}{1 + j\tau\omega}$

$e(t) = E_0 \sin(\omega t + \phi) \Rightarrow \underline{e} = E_0 e^{j(\omega t + \phi)}$

$$\Rightarrow \underline{u} = \frac{E_0 e^{j(\omega t + \phi)}}{2(1 + j\tau\omega)}$$

$$\Rightarrow \boxed{U_0 = |\underline{u}| = \frac{E_0}{2\sqrt{1 + (\omega\tau)^2}}}$$

4°/ $\psi = \arg\left(\frac{e^{j\phi}}{1 + \omega\tau}\right) = \phi - \arctan \omega\tau$

5°/ Solution générale : $u = \underbrace{u_H}_{\text{solution homogène}} + u_P \leftarrow$ solution parti

solution homogène

$$u + \tau\frac{du}{dt} = 0$$

$$1 + r\tau = 0 \Rightarrow r_1 = -\frac{1}{\tau}$$

$$u_H = Ae^{-r_1 t}$$

$$U_H = Ae^{-r_1 t} = Ae^{-t/\tau}$$

donc $u(t) = \underbrace{U_0 . \cos(\omega t + \psi)}_{u_P} + \underbrace{Ae^{-t/\tau}}_{u_H}$

$$u(t) = A e^{-t/\tau} + U_0 \cos(\omega t + \varphi)$$

– comme $u(0) = 0 \Rightarrow u(0) = A + U_0 \cos\varphi = 0$

$$\Rightarrow \boxed{A = -U_0 \cdot \cos\varphi}$$

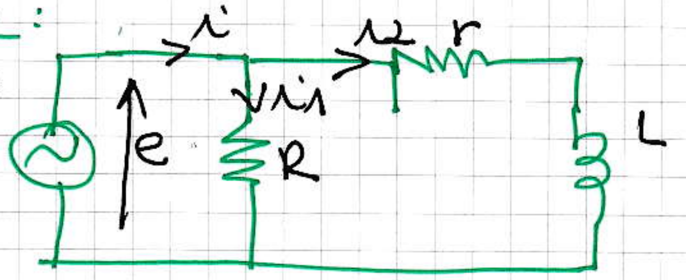
et si $\varphi = \pi/2 \Rightarrow A = 0 \Rightarrow$ on a le régime permanent $\Rightarrow \boxed{\Phi = \frac{\pi}{2} + \arctan \omega\tau}$

Exercice 2 :

$e(t) = E\sqrt{2} \sin\omega t$

$E = 375\,V$

$F = 50\,Hz$



1°/

$$\underline{I} = \underline{I_1} + \underline{I_2} \qquad \underline{I} = I e^{j\varphi}$$

$$\underline{I_1} = I_1 e^{j\varphi_1} \; ; \; \underline{I_2} = I_2 e^{j\varphi_2}$$

R est une résistance pure $\Rightarrow \varphi_1 = 0$

$$\underline{I} = I_1 + I_2 e^{j\varphi_2}.$$

$$\underline{I} = I_1 + I_2 \cos\varphi_2 + j I_2 \sin\varphi_2$$

$$I = \sqrt{(I_1 + I_2\cos\varphi_2)^2 + (I_2 \sin\varphi_2)^2}$$

$$= \sqrt{I_1^2 + I_2^2 \cos^2\varphi_2 + 2 I_1 I_2 \cos\varphi_2 + I_2^2 \sin^2\varphi_2}$$

$$I = \sqrt{I_1^2 + I_2^2 (\underbrace{\cos^2\varphi_2 + \sin^2\varphi_2}_{1}) + 2 I_1 I_2 \cos\varphi_2}$$

finalement : $\boxed{I = \sqrt{I_1^2 + I_2^2 + 2 I_1 I_2 \cos\varphi_2}}$

2°/ $P_M = r I_2^2$ mais I_2 inconnu

$$P_M = e \cdot I_2 \cdot \cos\varphi_2$$

on a $I^2 = I_1^2 + I_2^2 + 2 I_1 I_2 \cos\varphi_2$

$$\Rightarrow I_2 \cos\varphi_2 = \left(I^2 - (I_1^2 + I_2^2)\right) / (I_1 \times 2)$$

donc $P_m = E \cdot \frac{I^2 - I_1^2 - I_2^2}{2 I_1}$ ← $I_2 \cos\varphi_2$

$$I_1 = \frac{E}{R} = \frac{375}{36} = 10{,}42\ A$$

donc $P_m = 375\ \frac{16^2 - 10{,}42^2 - 7^2}{2 \times 10{,}42} = 1771\ W$

Remarque
↳ $r = \frac{P_m}{I_2^2} = \frac{1771}{7^2} =$ $\boxed{r = 36\,\Omega}$

3°/ $P_G = E \times I \cos\varphi$

$$\underline{I} = \underline{I_1} + \underline{I_2}$$

$$\underline{I} = I_1 + I_2 e^{j\varphi_2} = I_1 + I_2 \cos\varphi_2 + j I_2 \sin\varphi_2$$

$$I e^{j\varphi} = I_1 + I_2 \cos\varphi_2 + j I_2 \sin\varphi_2$$

$$I \cos\varphi + j I \sin\varphi = I_1 + I_2 \cos\varphi_2 + j I_2 \sin\varphi_2$$

égalité des parties réelles :

$$I \cos\varphi = I_1 + I_2 \cos\varphi_2$$

on a vu que $I_2 \cos\varphi_2 = \frac{I^2 - I_1^2 - I_2^2}{2 I_1}$

donc $I \cos\varphi = I_1 + \frac{I^2 - I_1^2 - I_2^2}{2 I_1}$

$$I \cos\varphi = \frac{2 I_1^2 + I^2 - I_1^2 - I_2^2}{2 I_1} = \frac{I^2 + I_1^2 - I_2^2}{2 I_1}$$

donc $\boxed{P_G = E I \cos\varphi = E \cdot \frac{(I^2 + I_1^2 - I_2^2)}{2 I_1}}$

AN : $P_G = 375\ \frac{16^2 + 10{,}42^2 - 7^2}{2 \times 10{,}42} =$

$$\boxed{P_G = 5679\ W}$$

4°) on a vu que :

$$I\cos\varphi = \frac{I^2 + I_1^2 - I_2^2}{2 I_1}$$

$$\Longrightarrow \cos\varphi = \frac{I^2 + I_1^2 - I_2^2}{2 I . I_1} = \frac{16^2 + 10{,}42^2 - 7^2}{2 \times 16 \times 10{,}42}$$

$$\boxed{\cos\varphi = 0{,}946}$$

Autre méthode : $\cos\varphi = \frac{P_G}{S} = \frac{P_G}{E \times I}$

$$\cos\varphi = \frac{5679}{375 \times 16} = 0{,}946$$

5°) $\cos\varphi' = 1 \Longrightarrow C$?

on place le condensateur en dérivation

$$\Longrightarrow i = i_1 + i_2 + i_c$$

$$\underline{i}_c = \frac{\underline{E}}{\underline{Z}_c} = \frac{\underline{E}}{\frac{1}{jC\omega}} = jC\omega\underline{E}$$

$$\Longrightarrow \underline{i} = \underline{i}_1 + \underline{i}_2 + \underline{i}_c$$

$$I e^{j\varphi'} = I_1 e^{j\varphi_1} + I_2 e^{j\varphi_2} + I_c e^{j\pi/2}$$

$\varphi_1 = 0$ car R_1 : résistance

$$I e^{j\varphi'} = I_1 + I_2 e^{j\varphi_2} + C\omega E e^{j\pi/2}$$

$$I\cos\varphi' + jI\sin\varphi' = I_1 + I_2\cos\varphi_2 + jI_2\sin\varphi_2 + jC\omega E$$

$$I\cos\varphi' + jI\underline{\sin\varphi'} = I_1 + I_2\cos\varphi_2 + j(\underline{I_2\sin\varphi_2 + C\omega E})$$

$\cos\varphi' = 1 \Longrightarrow \varphi' = 0$

donc la partie imaginaire est nulle

$$\Longrightarrow I_2\sin\varphi_2 + C\omega E = 0$$

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$$\Rightarrow C = -\frac{I_2 \sin\varphi_2}{\omega E}$$

$\cos\varphi_2 = \frac{I\cos\varphi - I_1}{I_2}$ et $\sin\varphi_2 < 0$.

$$= \frac{16 \times 0{,}946 - 10{,}42}{7} = 0{,}674$$

$\varphi_2 = -0{,}831$ rad

$$C = -\frac{7 \times \sin(-0{,}831)}{314 \times 375} = 43{,}9\,\mu F$$

<u>Autre méthode de Boucherot</u>

$\cos\varphi = 0{,}946 \longrightarrow$ on veut $\cos\varphi' = 1$

$\Rightarrow$ Nouvelle puissance réactive ($\varphi' = 0$)

$Q' = P_G \times \tan\varphi' = P_G \times \tan(0) = \underline{\underline{0\,\text{var}}}$

← Nouvelle Puissance

$Q' = Q + Q_C \Rightarrow Q_C = Q' - Q = 0 - Q$

ancienne

$\boxed{Q_C = -Q}$ Q_C : capacité réactive des capacité

$Q = P_G \times \tan\varphi = 5679 \times \tan\cos^{-1}(0{,}946)$

$Q = 1946\,\text{var}$

$Q_C = -Q = -1946 = -E^2 . C\omega$

$$\Rightarrow C = \frac{-Q_C}{E^2 . \omega} = -\frac{(-1946)}{375^2 \times 314}$$

Puissance réactive d'un condensateur soumis à une

$\boxed{C = 44\,\mu F}$

<u>de tension de valeur efficace E :</u>

$\boxed{Q_C = -E^2 \times C \times \omega}$