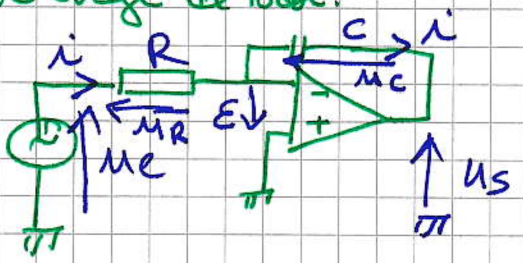
Correction TD: intégrateur

1°) Montage de base:



1-1

$$u_e - u_R + \varepsilon = 0$$

ε = 0V car l'ALI fonctionne en régime linéaire (présence d'une liaison électrique entre la sortie S et l'entrée inverseuse (−) : contre-réaction négative

donc $u_e - u_R = 0$

$u_e = Ri$ (1')

$u_s + u_c = 0$

$\Rightarrow u_s = -u_c$

$$i = C\frac{du_c}{dt} = -C\frac{du_s}{dt} \quad (2)$$

(2') dans (1') $\Longrightarrow$ $\boxed{u_e = -RC\frac{du_s}{dt}}$

ou $\boxed{\frac{du_s}{dt} = -\frac{1}{RC}\times u_e}$

1-2 à $t=0$ $u_c(0)=0$

$$u_e = U_0 + U_{em}\cos\omega t = 0{,}2 + 10\cos\omega t$$

Pour $u_e = U_0 = 0{,}2\,V$

$$\frac{du_s}{dt} = -\frac{1}{RC}.U_0 \Longrightarrow u_s(t) = -\frac{1}{RC}U_0 t + cte$$

à $t=0$ $u_c(0)=0 \Longrightarrow u_s=0 \Longrightarrow cte=0$

$$\Longrightarrow \boxed{u_s(t) = -\frac{1}{RC}.U_0.t}$$

$$u_s = -\frac{1}{10^3\times 100\,10^{-9}}\times 0{,}2\,t = -200t$$

la tension u_s décroît au bout d'un temps t_1 l'ALI se sature $\Longrightarrow -200t_1 = -12 \Longrightarrow$

$$t_1 = \frac{-12}{-200} = 0{,}06\,s$$

1-3 : Pour $u_e(t) = V_{em} \cos\omega t = 10 \cos\omega t$

$$\frac{du_s}{dt} = -\frac{1}{RC} \times V_{em} \cos\omega t$$

$$u_s(t) = -\frac{V_{em}}{RC\omega} \cdot \sin\omega t = \frac{-10}{10 \cdot 10^3 \times 100 \cdot 10^{-9} \times} \sin\omega t$$

$$\boxed{u_s(t) = -1{,}59 \sin\omega t}$$

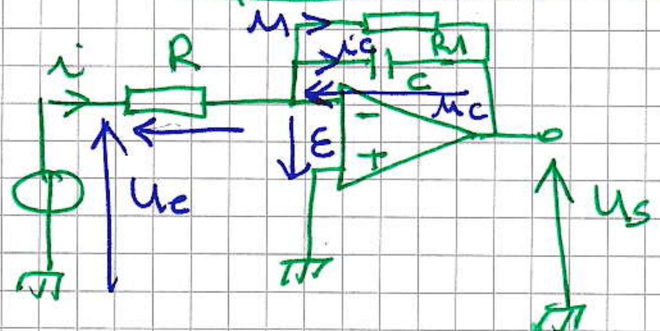
1-4 $u_e(t) = U_0 + V_{em} \cos\omega t$

on applique le théorème de superposition

donc $u_s(t) = -\frac{U_0}{RC} \cdot t - \frac{V_{em}}{RC\omega} \sin\omega t$

$$\boxed{u_s(t) = -200t - 1{,}59 \sin\omega t}$$

2°/



2-1 $U_e - Ri + \underset{0}{\varepsilon} = 0 \implies u_e - Ri = 0$

$u_s + u_c + \underset{0}{\varepsilon} = 0 \implies u_s + u_c = 0$

$i = i_c + i_1 \qquad C\frac{du_c}{dt} = -C\frac{du_s}{dt}$

$$i = i_C + i_1$$

$$\frac{u_e}{R} = -C\frac{du_s}{dt} - \frac{u_s}{R_1}$$

$$\boxed{R_1 C \frac{du_s}{dt} + u_s = -\frac{R_1}{R} \cdot u_e}$$

2-2 Pour $u_e = U_0$

$$R_1 C \frac{du_s}{dt} + u_s = -\frac{R_1}{R} U_0 \qquad \text{ED 1}^{\text{er}} \text{ ordre}$$

$$\tau = R_1 C = 10 \times 10.10^{3} \times 100.10^{-9} = 0{,}01\,s$$

$$u_s(t) = A e^{-t/\tau} + B$$

à $t = 0$ $\quad u_s = -u_c = 0$

$$u_s(0) = A + B = 0 \Longrightarrow A = -B$$

$\left(\text{Pour } t \to +\infty \quad \frac{du_s}{dt} = 0 \Longrightarrow u_s = -\frac{R_1 U_0}{R}\right)$ $B = -A = -\frac{R_1 U_0}{R}$

$$\Longrightarrow \boxed{U_s = -\frac{R_1}{R} U_0 \left(1 - e^{-t/\tau}\right)}$$

Pour $t = 5\tau = 5 \times 0{,}01 = 50\,ms$

$$u_s \simeq -\frac{R_1}{R} U_0 = -10\, U_0 = -2\,V$$

2-3 $u_e = U_{em} \cos wt$

on a $\tau \frac{du_s}{dt} + u_s = -\frac{R_1}{R} U_{em} \cos wt$

$$\frac{du_s}{dt} + \frac{1}{\tau} u_s = -\frac{U_{em}}{RC} \cos wt$$

$$\frac{du_s}{dt} = -\left(\frac{u_s}{\tau} + \frac{U_{em}}{RC} \cos wt\right)$$

$$\frac{du_s}{dt} = -\frac{1}{RC}\left(\frac{R}{R_1} u_s + U_{em} \cos wt\right)$$

on a $\frac{R_1}{R} = 10 \Longrightarrow \frac{R}{R_1} u_s < U_{em} \cos wt$

$$\Rightarrow \frac{du_s}{dt} = -\frac{U_{em}}{RC}\cos\omega t$$

$$\Rightarrow u_s(t) = \int -\frac{U_{em}}{RC}\cos\omega t \, dt$$

$$u_s(t) = -\frac{U_{em}}{RC\omega}.\sin\omega t + \underbrace{K}_{u_c=0}$$

$u_c(0) = u_s(0) = 0$

$$\boxed{u_s(t) = -\frac{U_{em}}{RC\omega}.\sin\omega t}$$

$$u_s(t) = -\frac{10}{10.10^3\times 100\times 2\pi\times 10^3}\sin\omega t$$

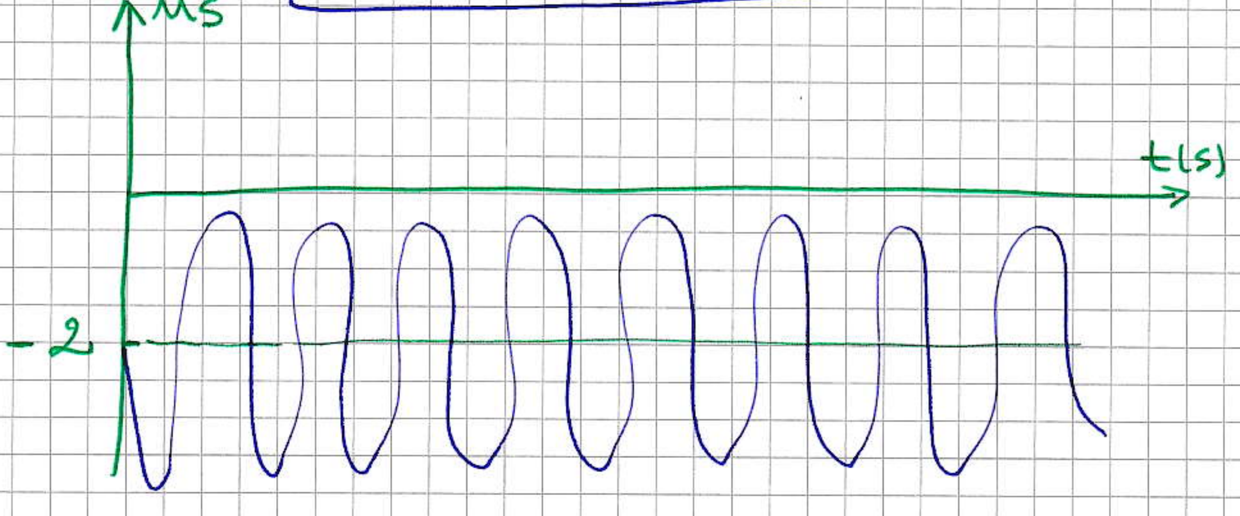
$$\boxed{u_s(t) = -1{,}59\sin\omega t}$$

<u>2-4</u> $u_e = U_0 + U_{em}\cos\omega t$

on applique le théorème du superposition

$$u_s(t) = -\frac{R_1 U_0}{R} - \frac{U_{em}}{RC\omega}.\sin\omega t$$

$$\boxed{u_s(t) = -2 - 1{,}59\sin\omega t}$$



dans ce cas l'ALI ne sature pas.