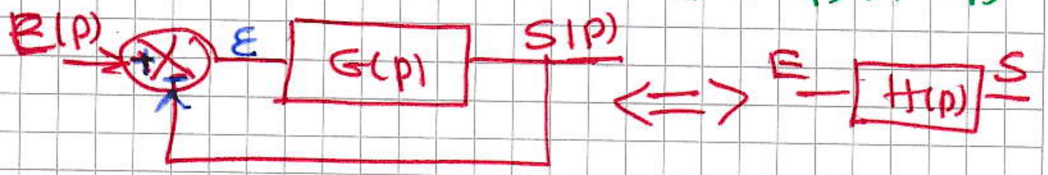
Correction TD1 : SA

Exercice 1 :

FTBO $G(p) = \dfrac{100}{(1+10p)(10+p)}$



$$H(p) = \frac{G(p)}{1+G(p)} = \frac{100/(\ \ )(\ \ )}{1+\frac{100}{(\ \ )(\ \ )}}$$

$$H(p) = \frac{100}{(1+10p)(10+p)+100}$$

$$H(p) = \frac{100}{10p^2+101p+110} = \frac{100}{p^2+10{,}1p+11}$$

– calcul de l'erreur $\varepsilon(p)$ en fonction de $G(p)$

$$\varepsilon(p) = E(p) - \underbrace{S(p)}_{\varepsilon(p)\times G(p)} = E(p) - \varepsilon(p)\,G(p)$$

$$\varepsilon(p)\,(1+G(p)) = E(p) \underset{(1)}{\Longrightarrow} \boxed{\varepsilon(p) = \frac{E(p)}{1+G(p)}}$$

l'erreur $\varepsilon(p)$ en fonction de la FTBO

– calcul de l'erreur $\varepsilon(p)$ en fonction de $H(p)$:

$$\varepsilon(p) = E(p) - \underbrace{S(p)}_{E(p)\times H(p)} = E(p) - E(p).H(p)$$

$$\Longrightarrow (2)\ \boxed{\varepsilon(p) = E(p)\,(1-H(p))}$$

l'erreur de $\varepsilon(p)$ en fct de la FTBF.

Pour l'erreur de position ou statique on a $E(p) = \frac{1}{p}$

〃 〃 de vitesse ou dynamique 〃 $E(p) = \frac{1}{p^2}$

erreur de position

$$\varepsilon_s = \lim_{p\to 0} p\,\varepsilon(p) = \lim_{p\to 0} p\times\frac{E(p)}{1+G(p)} = \lim_{p\to 0} p\times\frac{1/p}{1+G(p)}$$

$$\varepsilon_s = \lim_{p\to 0}\frac{1}{1+G(p)} = \frac{1}{1+G(0)} = \frac{1}{1+\frac{100}{10}} = \frac{1}{11}$$

$\varepsilon_s = 9{,}09\,\%$

erreur de vitesse

$$\varepsilon_V = \lim_{p \to 0} p\,\varepsilon(p) \quad \text{avec } E(p) = \frac{1}{p^2}$$

$$\varepsilon_V = \lim_{p \to 0} p \times \frac{E(p)}{1+G(p)} = \lim_{p \to 0} p \times \frac{1/p^2}{1+G(p)}$$

$$\varepsilon_V = \lim_{p \to 0} \frac{1}{p} \times \frac{1}{1+G(p)} = +\infty \qquad \boxed{\varepsilon_V = +\infty}$$

Remarque: on aurait pu utiliser la relation (2)

Exercice 2

FTBO: $G(p) = \frac{10}{p(1+p)(1+10p)}$

erreur de position (ou statique)

$$\varepsilon_S = \lim_{p \to 0} p\,\varepsilon(p) = \lim_{p \to 0} p\,\frac{E(p)}{1+G(p)}$$

avec $E(p) = \frac{1}{p}$

$$\varepsilon_S = \lim_{p \to 0} p\,\frac{1/p}{1+G(p)} = \lim_{p \to 0} \frac{1}{1+G(p)}$$

$$\varepsilon_S = \lim_{p \to 0} \frac{1}{1+\frac{10}{p(p+1)(1+10p)}} = 0$$

$$\boxed{\varepsilon_S = 0}$$

c'est un résultat attendu puisqu'il y a présence d'un intégrateur $\frac{1}{p}$ dans la FTBO.

erreur de vitesse (ou dynamique)

$$\varepsilon_V = \lim_{p \to 0} p\,\varepsilon(p) \quad \text{avec } E(p) = \frac{1}{p^2}$$

$$\varepsilon_V = \lim_{p \to 0} p\,\frac{E(p)}{1+G(p)} = \lim_{p \to 0} p \times \frac{1/p^2}{1+G(p)}$$

$$\varepsilon_V = \lim_{p \to 0} \frac{1}{p\left(1+\frac{10}{p(p+1)(1+10p)}\right)}$$

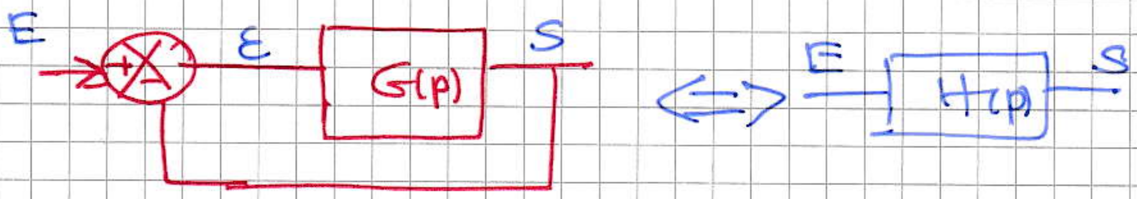
$$\varepsilon_V = \lim_{p \to 0} \frac{1}{p+\frac{10}{(p+1)(1+10p)}} = \frac{1}{0+\frac{10}{1}}$$

$$\boxed{\varepsilon_V = \frac{1}{10} = 10\%}$$

Exercice 3

FTBO : $G(p) = \frac{K}{(p+3)^2}$

$K > 0$.



$K\,? \Rightarrow \varepsilon_s = 5\,\%$

$$\varepsilon_s = \lim_{p \to 0} p\,\varepsilon(p) = \lim_{p \to 0} p\,\frac{E(p)}{1+G(p)}$$

avec $E(p) = 1/p$

$$\Rightarrow \varepsilon_s = \lim_{p \to 0} p\,\frac{1/p}{1+\frac{K}{(p+3)^2}} = \lim_{p \to 0} \frac{1}{1+\frac{K}{(p+3)^2}}$$

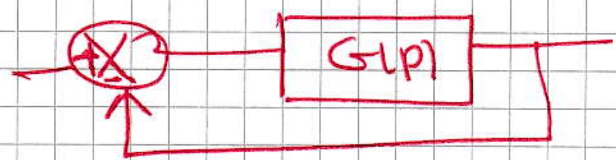
$$\varepsilon_s = \frac{1}{1+\frac{K}{9}} = \frac{9}{K+9} = \frac{5}{100}$$

$$\Rightarrow 5K + 45 = 900 \Rightarrow K = \frac{900-45}{5}$$

$$K = \frac{855}{5} = 171 \qquad \boxed{K = 171}$$

Exercice 4

FTBO : $G(p) = \frac{2{,}1}{p(1+0{,}11p)}$



a) $$H(p) = \frac{G(p)}{1+G(p)} = \frac{2{,}1}{2{,}1 + p(1+0{,}11p)}$$

$$H(p) = \frac{2{,}1}{0{,}11p^2 + p + 2{,}1} = \frac{2{,}1/0{,}11}{p^2 + \frac{p}{0{,}11} + \frac{2{,}1}{0{,}11}}$$

$$H(p) = \frac{19}{p^2 + 9p + 19} = \frac{K_0}{p^2 + 2m\omega_0 p + \omega_0^2}$$

$\omega_0^2 = 19 \Rightarrow \omega_0 = \sqrt{19}$ rad/s

$$\boxed{\omega_0 = 4{,}36 \text{ rad s}^{-1}}$$

b) coefficient d'amortissement

$2m\omega_0 = 9 \Rightarrow m = \dfrac{9}{2\omega_0}$

$m = \dfrac{9}{2 \times 4,36} = 1,032$

c) $m > 1 \Rightarrow$ pas de dépassement.

Il y a dépassement si $0 < m < 1$

d) $\varepsilon_s = \lim\limits_{p \to 0} p\,\varepsilon(p) = \lim\limits_{p \to 0} p \dfrac{E(p)}{1 + G(p)}$

$E(p) = 1/p$

$\varepsilon_s = \lim\limits_{p \to 0} \dfrac{1}{1 + G(p)} = \lim\limits_{p \to 0} \dfrac{1}{1 + \dfrac{21}{p(1+0{,}1p)}}$

$\boxed{\varepsilon_s = 0}$ évident il y a présence d'un intégrateur dans la chaîne directe

e) erreur de vitesse :

$\varepsilon_v = \lim\limits_{p \to 0} p\,\varepsilon(p)$ avec $E(p) = 1/p^2$

$\varepsilon_v = \lim\limits_{p \to 0} p \dfrac{E(p)}{1 + G(p)} = \lim\limits_{p \to 0} \dfrac{1}{p\left(1 + \dfrac{21}{p(1+0{,}1p)}\right)}$

$\varepsilon_v = \lim\limits_{p \to 0} \dfrac{1}{p + \dfrac{21}{1+0{,}1p}} = \dfrac{1}{21} = 0,476$

$\boxed{\varepsilon_v = 47{,}6\,\%}$

f) $\varepsilon_a = \lim\limits_{p \to 0} p\,\varepsilon(p) = \lim\limits_{p \to 0} \dfrac{E(p)}{1 + G(p)}$

avec $E(p) = 1/p^3$

$\varepsilon_a = \lim\limits_{p \to 0} p \dfrac{1/p^3}{1 + G(p)} = \lim \dfrac{1}{p^2\left(1 + \dfrac{21}{p(1+0{,}1p)}\right)}$

$\boxed{\varepsilon_a = \infty}$

Exercice 5

FTBO : $G(p) = \dfrac{K}{(1+p)(1+2p)}$

2°/ $H(p) = \dfrac{G(p)}{1+G(p)}$

$$H(p) = \frac{\dfrac{K}{(1+p)(1+2p)}}{1+\dfrac{K}{(1+p)(1+2p)}} = \frac{K}{(1+p)(1+2p)+K}$$

$$\boxed{H(p) = \frac{K}{2p^2+3p+K+1}}$$

$$H(p) = \frac{K/(K+1)}{1+\dfrac{3}{K+1}p+\dfrac{p^2}{\left(\frac{K+1}{2}\right)}} = \frac{K_0}{1+\dfrac{2m}{\omega_0}p+\dfrac{p^2}{\omega_0^2}}$$

un dépassement de 20% $\Longrightarrow$ un coefficient d'amortissement $\boxed{m = 0{,}45}$

$$\frac{2m}{\omega_0} = \frac{3}{K+1} \Longrightarrow m = \frac{3\omega_0}{2(K+1)}$$

$$\omega_0^2 = \frac{K+1}{2} \Longrightarrow \omega_0 = \sqrt{\frac{K+1}{2}}$$

$$\Longrightarrow m = \frac{3}{2} \cdot \frac{\sqrt{\frac{K+1}{2}}}{K+1} = \frac{3}{2}\,\frac{1}{\sqrt{2}\sqrt{K+1}}$$

$$\Longrightarrow \sqrt{K+1} = \frac{3}{2\sqrt{2}\times m}$$

$$\boxed{K = \left(\frac{3}{2\sqrt{2}\,m}\right)^2 - 1}$$

AN : $K = \dfrac{9}{4\times 2\times 0{,}45^2} - 1 = 4{,}56$

$$\boxed{K = 4{,}56}$$

b) <u>erreur de position</u>

$$\varepsilon_s = \lim_{p \to 0} p \times \varepsilon(p) = \lim_{p \to 0} p \frac{E(p)}{1+G(p)}$$

$$E(p) = 1/p$$

$$\varepsilon_s = \lim_{p \to 0} \frac{1/p}{p(1+G(p))} = \lim_{p \to 0} \frac{1}{1+G(p)}$$

$$\varepsilon_s = \frac{1}{1+G(0)} = \frac{1}{1+\frac{K}{1}} = \frac{1}{1+4}$$

$$\varepsilon_s = \frac{1}{5{,}55} = 0{,}179 \qquad \boxed{\varepsilon_s = 17{,}9\%}$$

<u>temps de réponse:</u>

d'après l'abaque : on a pour m = 0,46

le produit $\omega_0 \times tr = 5{,}2$

$$\Rightarrow tr = \frac{5{,}2}{\omega_0} \qquad \omega_0 = \sqrt{\frac{K+1}{2}}$$

$$\omega_0 = \sqrt{\frac{4{,}56+1}{2}} = 1{,}29 \text{ rad s}^{-1}$$

$$tr = \frac{5{,}2}{1{,}29} = 4\,s$$

$$\boxed{tr_{5\%} = 4\,s}$$