Correction exercice TZ inverse

a) $X(z) = \dfrac{1 + 2z^{-1} + 4z^{-2}}{1 + z^{-1}}$

$$X(z) = \frac{4z^{-2} + 2z^{-1} + 1}{1 + z^{-1}}$$

$$X(z) = \frac{4z^{-2} + \overbrace{4z^{-1} - 2z^{-1}}^{2z^{-1}} + \overbrace{3 - 2}^{1}}{1 + z^{-1}}$$

$$X(z) = \frac{4z^{-1}(z^{-1} + 1)}{z^{-1} + 1} - \frac{2(z^{-1} + 1)}{1 + z^{-1}} + \frac{3}{1 + z^{-1}}$$

$$X(z) = 4z^{-1} - 2 + \frac{3}{1 + z^{-1}}$$

$$\boxed{x(n) = 4\delta(n-1) - 2\delta(n) + 3 \cdot (-1)^n u(n)}$$

$$\frac{3}{1 + z^{-1}} = \frac{3}{1 - (-z)^{-1}} \xrightarrow{TZ^{-1}} (-1)^n u(n)$$

b) $X(z) = \dfrac{1 + 2z^{-1} + z^{-2}}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$

$$X(z) = \frac{z^2 + 2z + 1}{z^2 - \frac{3}{2}z + \frac{1}{2}} = \frac{z^2 + 2z + 1}{(z - \frac{1}{2})(z - 1)} = \frac{(z+1)^2}{(z-\frac{1}{2})(z-1)}$$

$$X(z) = A + \frac{Bz}{z - \frac{1}{2}} + \frac{Cz}{z - 1}$$

$$B = \left.\frac{(z - 1/2)}{z} \times X(z)\right|_{z=\frac{1}{2}} = \left.\frac{(z+1)^2}{z(z-1)}\right|_{z=\frac{1}{2}} = \frac{(3/2)^2}{\frac{1}{2} \times (-1/2)}$$

$$B = \frac{9/4}{-1/4} = -9$$

$$C = \left.\left(\frac{z-1}{z}\right) \cdot X(z)\right|_{z=+1} = \left.\frac{(z+1)^2}{z(z - \frac{1}{2})}\right|_{z=1} = \frac{2^2}{1 \times \frac{1}{2}} = 8$$

- Calcul du coefficient A:

$$\lim_{z \to +\infty} X(z) = 1 = A + B + C \Rightarrow A = 1 - (B+C)$$

$$A = 1 - \underbrace{(-9+8)}_{-1} = 2$$

donc $X(z) = 2 + \frac{8z}{z-1} - \frac{9z}{z-1/2}$

d'après la table on a

	$x(n)$	$X(z)$	
Dirac :	$\delta(n)$	$\longrightarrow 1$	*
Echelon :	$u(n)$	$\longrightarrow \frac{z}{z-1}$	*
	$n u(n)$	$\longrightarrow \frac{z}{(z-1)^2}$	
	$\delta(n-k)$	$\longrightarrow z^{-k}$	
	$a^n u(n)$	$\longrightarrow \frac{z}{z-a}$	*
	$n^2 . u(n)$	$\longrightarrow \frac{z(z+1)}{(z-1)^2}$	

ce qui donne :

$$2 \longrightarrow 2\delta n$$

$$\frac{8z}{z-1} \longrightarrow 8 \times 1^n = 8\,u(n)$$

$$-\frac{9z}{z-1/2} \longrightarrow -9\left(\frac{1}{2}\right)^n u(n)$$

donc $\boxed{x(n) = 2\delta(n) + \left(8 - 9\left(\frac{1}{2}\right)^n\right) u(n)}$