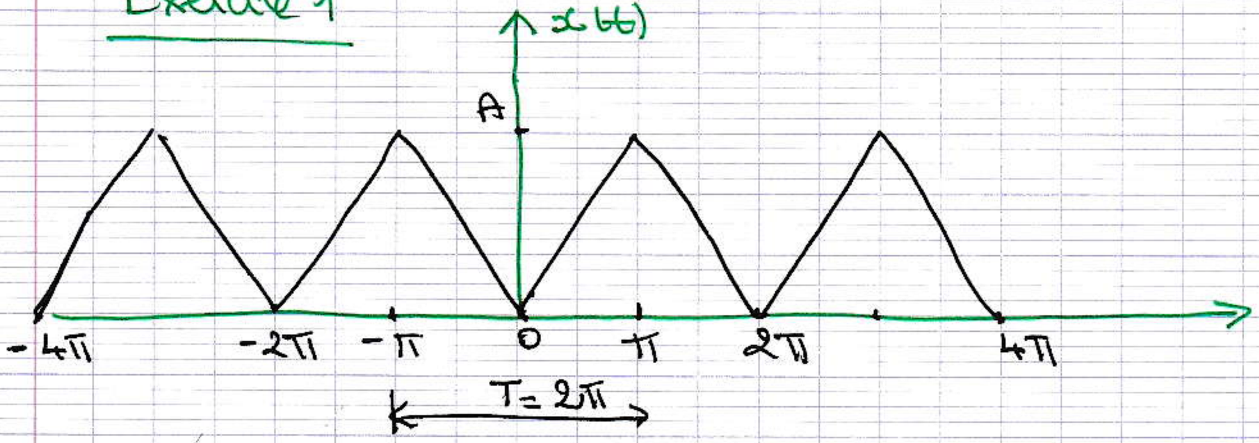
Correction

Exercice 1



on constate que le signal $x(t)$ est pair.

la pulsation $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1\ rad.s^{-1}$

$$x(t) = \begin{cases} -\frac{A}{\pi}t & -\pi \leq t \leq 0 \\ \frac{A}{\pi}.t & 0 \leq t \leq \pi \end{cases}$$

$x(t) = x(-t)$ donc

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{2}{T}\int_0^{T/2} x(t)\,dt$$

on a les $b_n = 0$ (car $x(t)$ paire)

$$a_n = \frac{2}{T}\int_0^T x(t).\cos n\omega_0 t\,dt = \frac{4}{T}\int_0^{T/2} x(t)\cos n\omega_0 t\,dt$$

Donc: $a_0 = \frac{2}{T}\int_0^{\pi} \frac{A}{\pi}t\,dt = \frac{2A}{2\pi\pi}\left[\frac{t^2}{2}\right]_0^{\pi}$

Donc $T = 2\pi$

$$a_0 = \frac{A}{\pi^2}\left(\frac{\pi^2}{2} - 0\right) = \frac{A}{2}$$

$\boxed{a_0 = \frac{A}{2}}$

$$a_n = \frac{\;}{2\pi}\int_0^{T/2} \frac{A}{\pi}.t.\cos n\omega_0 t\,dt$$

$\omega_0 = 1$

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$$a_n = \frac{2A}{\pi^2}\left[\int_0^{T/2} t.\cos nt \, dt\right]$$

$$a_n = \frac{2A}{\pi^2}\left[\int_0^{\pi} t.\cos nt \, dt\right]$$

$$a_n = \frac{2A}{\pi^2}\left[\left(\frac{t\sin nt}{n}\right)_0^{\pi} - \int_0^{\pi}\frac{\sin nt}{n}\,dt\right]$$

$$a_n = \frac{2A}{\pi^2}\left(\frac{\pi \sin n\pi}{n} - 0\right) + \frac{\cos nt}{n^2}\Big[\Big]_0^{\pi}$$

$$a_n = \frac{2A}{\pi^2}\left(\frac{\pi \sin(n\pi)}{n} + \frac{\cos n\pi - 1}{n^2}\right)$$

$\sin(n\pi) = 0 \quad \forall n \in \mathbb{N}$

$$\cos n\pi \begin{cases} = -1 & \text{si } n \text{ impair} \\ = +1 & \text{si } n \text{ pair} \end{cases}$$

donc $\cos(n\pi) = (-1)^n$

donc $a_n = \frac{2A}{n^2\pi^2}\left((-1)^n - 1\right)$

$$a_n = \begin{cases} \frac{2A}{n^2\pi^2} \times (-2) & \text{si } n \text{ impair} \\ 0 & \text{si } n \text{ pair} \end{cases}$$

$$a_n = \begin{cases} \frac{-4A}{n^2\pi^2} & \text{si } n \text{ impair} \\ 0 & \text{si } n \text{ pair} \end{cases}$$

donc $x(t) = a_0 + \sum_{n=1}^{+\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$

$$x(t) = \frac{A}{2} + \sum_{\substack{n=1 \\ n \text{ impair}}}^{+\infty} \frac{-4A}{n^2\pi^2} \cos nt$$

$$a_n = \frac{-4A}{n^2\pi^2} \quad \text{si } n \text{ impair}$$

on pose $n = 2k+1$ si $n=1$ $k=0$

si $n=1$ $k=3$

si $n=2$ $k=5$

donc $a_n = a_{2k+1} = \dfrac{-4A}{(2k+1)^2\pi^2}$

$0 < k < +\infty$

donc $\boxed{x(t) = \dfrac{A}{2} + \displaystyle\sum_{k=0}^{+\infty} \dfrac{-4A}{(2k+1)^2\pi^2} \cos((2k+1)t)}$

<u>Remarque:</u>

Pour intégrer $t \cdot \cos nt$ il faut utiliser l'intégration par partie

$(uv)' = u'v + uv' \Longrightarrow u'v = (uv)' - uv'$

$u' = \cos nt \Longrightarrow u = \dfrac{\sin nt}{n}$

$v = t \Longrightarrow v' = 1$