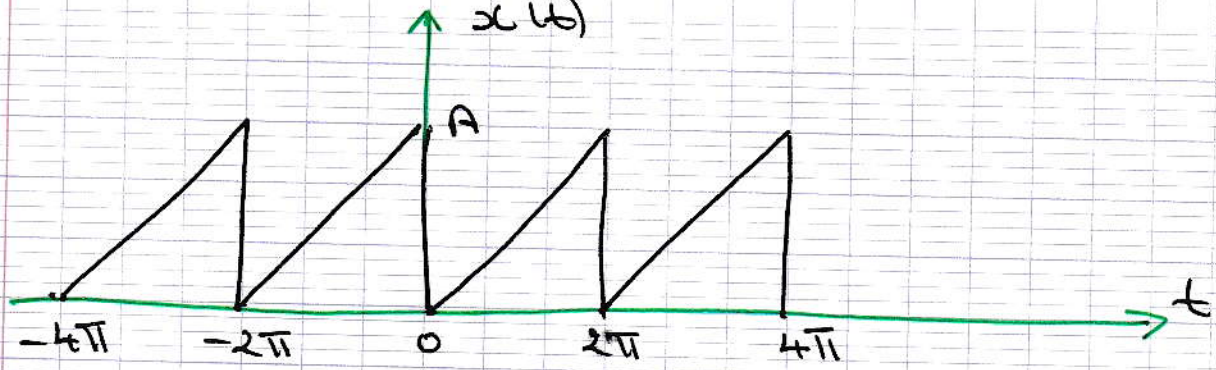

$$x(t) = \begin{cases} \frac{A}{2\pi} \cdot t & 0 < t < 2\pi \end{cases}$$

$$w_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1.$$

<u>La série de Fourier :</u>

$$x(t) = a_0 + \sum_{n=1}^{+\infty} a_n \cos(n w_0 t) + b_n \sin(n w_0 t)$$

$$x(t) = a_0 + \sum_{n=1}^{+\infty} a_n \cos(nt) + b_n \sin(nt)$$

* $$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{2\pi}\int_0^{2\pi} \frac{A}{2\pi} t\,dt$$

$$a_0 = \frac{A}{4\pi^2}\left[\frac{t^2}{2}\right|_0^{2\pi} = \frac{A}{8\pi^2}\left((2\pi)^2 - 0\right)$$

$$a_0 = \frac{A \times 4\pi^2}{8\pi^2} = A/2 \qquad \boxed{a_0 = \frac{A}{2}}$$

* $$a_n = \frac{2}{T}\int_0^T x(t) \cos nt\,dt = \frac{2}{2\pi}\int_0^{2\pi} \frac{A}{2\pi} t \cos nt\,dt$$

$$a_n = \frac{A}{2\pi^2}\int_0^{2\pi} t \cos nt\,dt$$

$$a_n = \frac{A}{2\pi^2}\left(\left[\frac{t \sin nt}{n}\right|_0^{2\pi} - \int_0^{2\pi} \frac{\sin nt}{n}\,dt\right]\right)$$

Intégration par partie $v = t$, $v' = 1$

$u' = \cos nt$

$u = \frac{\sin nt}{n}$

$$a_n = \frac{A}{2\pi^2}\left[0 - 0 - \frac{1}{n}\left(-\frac{\cos nt}{n}\right)\Big|_0^{2\pi}\right]$$

$$a_n = \frac{A}{2\pi^2}\left(+\frac{1}{n}(+1-1)\right) = 0$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{2}{T}\int_0^T x(t)\cdot \sin nt \, dt \qquad T = 2\pi \quad (\omega_0 = 1)$$

$$b_n = \frac{2}{T}\int_0^{2\pi} \frac{At}{2\pi}\cdot \sin nt \, dt = \frac{2A}{(2\pi)^2}\int_0^{2\pi} t \cdot \sin nt \, dt$$

$$b_n = \frac{A}{2\pi^2}\int_0^{2\pi} t \cdot \sin nt \, dt$$

$$b_n = \frac{A}{2\pi^2}\left(t \times \left(-\frac{\cos nt}{n}\right)\Big|_0^{2\pi} - \int_0^{2\pi} -\frac{\cos nt}{n} \, dt\right)$$

$$b_n = \frac{A}{2\pi^2}\left(\frac{-2\pi\cos 2n\pi - 0}{n} + \frac{1}{n}\cdot \frac{\sin nt}{n}\Big|_0^{2\pi}\right)$$

$$b_n = \frac{A}{2\pi^2}\left(-\frac{2\pi \times 1}{n} + \frac{1}{n}\frac{(0-0)}{n}\right)$$

$$b_n = \frac{A}{2\pi^2} \times \left(-\frac{2\pi}{n}\right) = -\frac{A}{\pi \cdot n} \qquad \boxed{b_n = \frac{-A}{\pi n}}$$

donc $x(t) = \frac{A}{2} + \sum_{n=1}^{+\infty} \frac{-A}{\pi n} \sin nt$

$$\boxed{x(t) = \frac{A}{2} - \sum_{n=1}^{+\infty} \frac{A}{\pi \cdot n} \sin nt} \qquad \boxed{0 \leq t \leq 2\pi}$$

$C_n = \sqrt{a_n^2 + b_n^2}$ et $\theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$

$$x(t) = C_0 + \sum_{n=1}^{+\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$C_n = \sqrt{0^2 + \left(-\frac{A}{\pi n}\right)^2} = \frac{A}{\pi n} \; ; \; C_0 = a_0 = \frac{A}{2}$$

$$\theta_n = +\tan^{-1}\left(\frac{-A/\pi n}{0}\right) = +\pi/2$$

donc $\boxed{x(t) = \frac{A}{2} + \sum_{n=1}^{+\infty} \frac{A}{n\pi} \cos\left(nt + \frac{\pi}{2}\right)}$

forme polaire