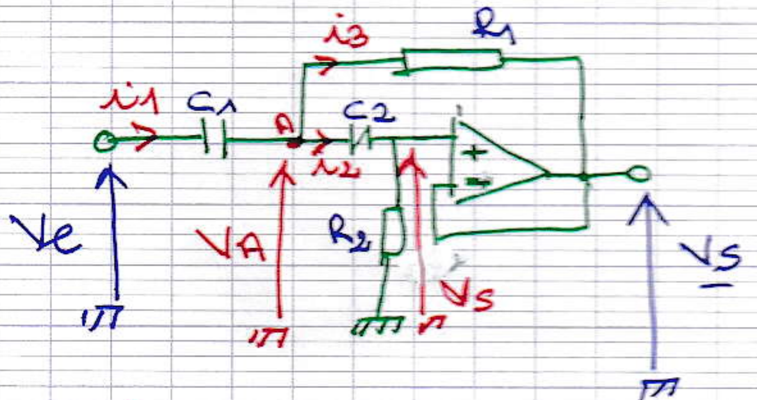
Correction TD. Filtre Sallen & Key



filtre passe-haut

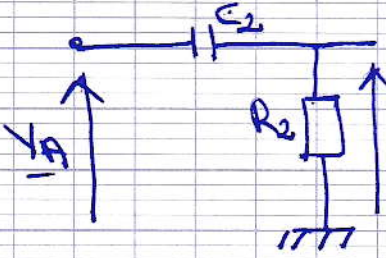
Loi des noeuds au pt A :

$i_1 = i_2 + i_3$ avec $\begin{cases} i_1 = \dfrac{\underline{V_e} - \underline{V_A}}{\underline{Z_{C1}}} \\ i_2 = \dfrac{\underline{V_A} - \underline{V_S}}{\underline{Z_{C2}}} \end{cases}$

et $i_3 = \dfrac{\underline{V_A} - \underline{V_S}}{R_1}$

donc $\dfrac{\underline{V_e} - \underline{V_A}}{\underline{Z_{C1}}} = \dfrac{\underline{V_A} - \underline{V_S}}{\underline{Z_{C2}}} + \dfrac{\underline{V_A} - \underline{V_S}}{R_1}$

(1) $\dfrac{\underline{V_e}}{\underline{Z_{C1}}} = \underline{V_A}\left(\dfrac{1}{\underline{Z_{C1}}} + \dfrac{1}{\underline{Z_{C2}}} + \dfrac{1}{R_1}\right) - \underline{V_S}\left(\dfrac{1}{\underline{Z_{C2}}} + \dfrac{1}{R_1}\right)$



$\underline{V_S} = \underline{V_A}\left(\dfrac{R_2}{\underline{Z_{C2}} + R_2}\right)$

$\Longrightarrow \underline{V_A} = \dfrac{\underline{Z_{C2}} + R_2}{R_2}\,\underline{V_S} = \left(1 + \dfrac{\underline{Z_{C2}}}{R_2}\right)\underline{V_S}$

$\boxed{\underline{V_A} = \left(1 + \dfrac{\underline{Z_{C2}}}{R_2}\right)\cdot \underline{V_S}}$ (3)

(3) dans (1) $\Longrightarrow$

$\dfrac{\underline{V_e}}{\underline{Z_{C1}}} = \left(1 + \dfrac{\underline{Z_{C2}}}{R_2}\right)\underline{V_S} \times \left(\dfrac{1}{\underline{Z_{C1}}} + \dfrac{1}{\underline{Z_{C2}}} + \dfrac{1}{R_1}\right) - \underline{V_S}\left(\dfrac{1}{\underline{Z_{C2}}} + \dfrac{1}{R_1}\right)$

$$\frac{\underline{V_e}}{\underline{Z_{C_1}}} = \underline{V_s}\left[\frac{1}{\underline{Z_{C_1}}} + \cancel{\frac{1}{\underline{Z_{C_2}}}} + \cancel{\frac{1}{R_1}} + \frac{\underline{Z_{C_2}}}{\underline{Z_{C_1}} R_2} + \frac{1}{R_2} + \frac{\underline{Z_{C_2}}}{R_1 R_2} - \cancel{\frac{1}{\underline{Z_{C_2}}}} - \cancel{\frac{1}{R_1}}\right]$$

$$\frac{\underline{V_e}}{\underline{Z_{C_1}}} = \underline{V_s}\left[\frac{1}{\underline{Z_{C_1}}} + \frac{\underline{Z_{C_2}}}{\underline{Z_{C_1}} R_2} + \frac{1}{R_2} + \frac{\underline{Z_{C_2}}}{R_1 R_2}\right]$$

$$T_1(j\omega) = \frac{\underline{V_s}}{\underline{V_e}} = \frac{1}{Z_{C_1}\left[\frac{1}{Z_{C_1}} + \frac{Z_{C_2}}{Z_{C_1} R_2} + \frac{1}{R_2} + \frac{Z_{C_2}}{R_1 R_2}\right]}$$

$$T_1(j\omega) = \frac{1}{1 + \frac{\underline{Z_{C_2}}}{R_2} + \frac{Z_{C_1}}{R_2} + \frac{Z_{C_1} Z_{C_2}}{R_1 R_2}}$$

$$T_1(j\omega) = \frac{1}{1 + \frac{Z_{C_1} + Z_{C_2}}{R_2} + \frac{Z_{C_1} Z_{C_2}}{R_1 R_2}}$$

$$\underline{Z_{C_1}} = \frac{1}{jC_1\omega} \; ; \quad \underline{Z_{C_2}} = \frac{1}{jC_2\omega}$$

$$T_1(j\omega) = \frac{1}{1 + \frac{1}{jR_2C_1\omega} + \frac{1}{jR_2C_2\omega} + \frac{1}{j^2 R_1R_2C_1C_2\omega^2}}$$

$$T_1(j\omega) = \frac{1}{1 + \frac{1}{jR_2C_1\omega} + \frac{1}{jR_2C_2\omega} + \frac{1}{j^2 R_1R_2C_1C_2\omega^2}}$$

$$T_1(j\omega) = \frac{1}{1 + \frac{1}{jR_2\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) + \frac{1}{j^2 R_1R_1C_1C_2\omega^2}}$$

3

$R_1 = R_2 = R$ et $C_1 = C_2 = C$

$$T_1(j\omega) = \frac{1}{1 + \frac{2}{jRC\omega} + \frac{1}{(jRC\omega)^2}}$$

$$T_1(j\omega) = \frac{(jRC\omega)^2}{(jRC\omega)^2\left(1+\frac{2}{jRC\omega}+\frac{1}{(jRC\omega)^2}\right)}$$

$$T_1(j\omega) = \frac{\left(j\frac{\omega}{\omega_0}\right)^2}{(jRC\omega)^2 + 2jRC\omega + 1}$$

$$T_1(j\omega) = \frac{\left(j\frac{\omega}{\omega_0}\right)}{1 + 2j\left(\frac{\omega}{\omega_0}\right) + \left(j\frac{\omega}{\omega_0}\right)^2}$$

avec $\boxed{\omega_0 = \frac{1}{RC}}$

forme canonique d'un filtre passe haut de 2nd ordre

$$T(j\omega) = \frac{A\left(j\frac{\omega}{\omega_0}\right)^2}{1 + 2mj\frac{\omega}{\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2}$$

$2m = 2 \Rightarrow \boxed{m = 1}$; $\boxed{A = 1}$

$$\omega_0 = \frac{1}{RC} = \frac{1}{10 \cdot 10^3 \times 10 \cdot 10^{-9}}$$

$\omega_0 = 10^4$ rd s^{-1}

$f_0 = \frac{\omega_0}{2\pi} = 1590$ Hz