

Correction

a) SLI causal: $y(n) - \frac{7}{6}y(n-1] + \frac{1}{3}y(n-2) = x(n)$ (1)

on passe à la TZ:

$$Y(z) - \frac{7}{6}z^{-1}Y(z) + \frac{1}{3}z^{-2}Y(z) = X(z)$$

$$Y(z) \left[1 - \frac{7}{6}z^{-1} + \frac{1}{3}z^{-2} \right] = X(z)$$

la fonction de transfert $H(z) = \frac{Y(z)}{X(z)}$:

$$H(z) = \frac{1}{1 - \frac{7}{6}z^{-1} + \frac{1}{3}z^{-2}} = \frac{z^2}{z^2 - \frac{7}{6}z + \frac{1}{3}}$$

on factorise $z^2 - \frac{7}{6}z + \frac{1}{3} = (z - \frac{1}{2})(z - \frac{2}{3})$

$$H(z) = \frac{z^2}{(z - \frac{1}{2})(z - \frac{2}{3})}$$

avec $|z| > \frac{2}{3}$

b) Réponse impulsionnelle $h(n]$

$$h(n] = \mathcal{TZ}^{-1}(H(z))$$

on calcule $\frac{H(z)}{z} = \frac{z}{(z - \frac{1}{2})(z - \frac{2}{3})} = \frac{A}{z - \frac{1}{2}} + \frac{B}{z - \frac{2}{3}}$

$$A = (z - \frac{1}{2}) \cdot \frac{H(z)}{z} \Big|_{z = \frac{1}{2}} = \frac{z}{z - \frac{2}{3}} \Big|_{z = \frac{1}{2}} = \frac{1/2}{1/2 - 2/3} = -3$$

$$B = (z - \frac{2}{3}) \cdot \frac{H(z)}{z} \Big|_{z = \frac{2}{3}} = \frac{z}{z - \frac{1}{2}} \Big|_{z = \frac{2}{3}} = \frac{2/3}{2/3 - 1/2} = 4$$

Donc $\frac{H(z)}{z} = \frac{-3}{z - \frac{1}{2}} + \frac{4}{z - \frac{2}{3}}$

finallement $H(z) = -3 \times \frac{z}{z - \frac{1}{2}} + 4 \times \frac{z}{z - \frac{2}{3}}$

on pose d'o Tz inverse

$$h(n) = \left(-3 \left(\frac{1}{2} \right)^n + 4 \times \left(\frac{2}{3} \right)^n \right) u(n)$$

c) $Y(z) = X(z) \times H(z)$

Reponse indicielle $y(n) = ?$

$X(z) = \frac{z}{z-1}$ (l'entree du signal est un échelon)

$$\Rightarrow Y(z) = \left(\frac{z}{z-1} \right) \times \frac{z^2}{(z - \frac{1}{2})(z - \frac{2}{3})}$$

on isole $\frac{Y(z)}{z} = \frac{z^2}{(z-1)(z-\frac{1}{2})(z-\frac{2}{3})} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{2}} + \frac{C}{z-\frac{2}{3}}$

$$A = (z-1) \cdot \left. \frac{Y(z)}{z} \right|_{z=1} = \left. \frac{z^2}{(z-\frac{1}{2})(z-\frac{2}{3})} \right|_{z=1} = \frac{1^2}{(1-\frac{1}{2})(1-\frac{2}{3})} = 6$$

$$B = (z-\frac{1}{2}) \cdot \left. \frac{Y(z)}{z} \right|_{z=\frac{1}{2}} = \left. \frac{z^2}{(z-\frac{2}{3})(z-1)} \right|_{z=\frac{1}{2}} = \frac{(\frac{1}{2})^2}{(\frac{1}{2}-\frac{2}{3})(\frac{1}{2}-1)} = 3$$

$$C = (z-\frac{2}{3}) \cdot \left. \frac{Y(z)}{z} \right|_{z=\frac{2}{3}} = \left. \frac{z^2}{(z-1)(z-\frac{1}{2})} \right|_{z=\frac{2}{3}} = \frac{(\frac{2}{3})^2}{(\frac{2}{3}-1)(\frac{2}{3}-\frac{1}{2})} = -8$$

finallement: $\frac{Y(z)}{z} = \frac{6}{z-1} + \frac{3}{z-\frac{1}{2}} - \frac{8}{z-\frac{2}{3}}$

et $Y(z) = 6 \frac{z}{z-1} + 3 \frac{z}{z-\frac{1}{2}} - 8 \frac{z}{z-\frac{2}{3}}$

et $y(n) = 6 + 3 \cdot \left(\frac{1}{2} \right)^n - 8 \cdot \left(\frac{2}{3} \right)^n$